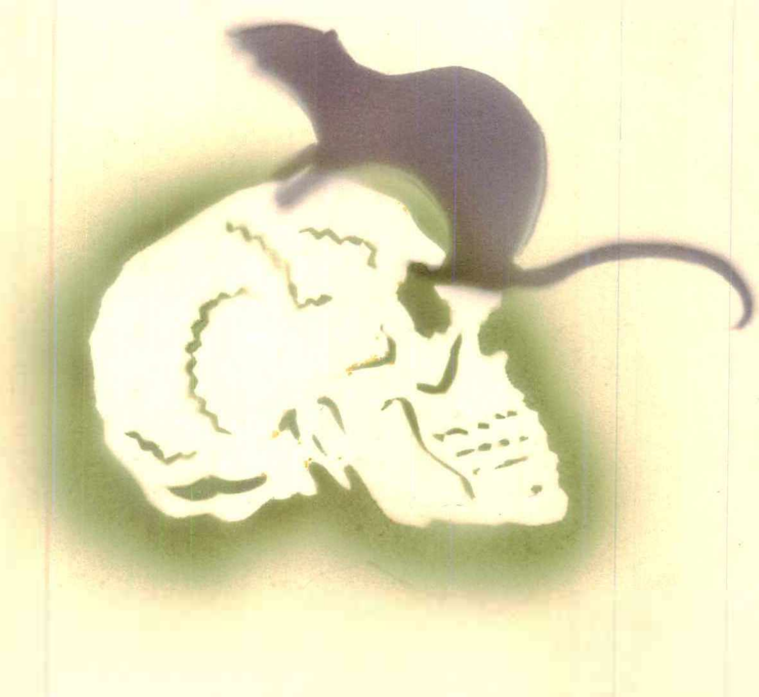
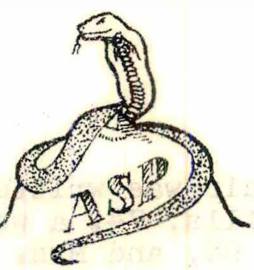
En GARDE



EN GARDE! A FAPA PUBLICATION WHOLE NUMBER XVI.



Fathered With Fannish Fecundity

By AL (ambitious) ASHLEY of

643 South Bixel Street,

Los Angeles 14, California.

JANUARY, 1946, MAILING.

YE MAPPE OF YE CONTENTE

CREDIT DEPARTMENT

THE FRONT COVER is	By Al Ashley.
AIRBRUSHING was done	By The Editor.
LITHOGRAPHED INSERT	By Lou Goldstone.
INTERIOR ARTWORK was secured	By Special Arrangement.

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DEBRIS DEPARTMENT

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AGED MAN KILLS WIFE, SELF AND "OTHER WOMAN"

The final chapter of a triangle love affair was written in Battle Creek, Mich., recently, when John H. Wills, 74, a wealthy retired business man murdered his wife, Ella, 68, and Mrs. Maggie M. Steward, 53, and then committed suicide.

Under the pretense of taking Mrs. Steward for a ride, Wills drove her to a remote spot six miles from the city, shot her in the head and then cut her throat with a razor. Upon returning home, he immediately shot his wife and then killed himself.

-----Weird Tales, October 1923 (Filler item

((Seems odd, somehow, to read this bit of hometown news in 1945, and from a 1923 issue of Weird Tales. And just what was it doing in Weird Tales in the first place? Fantastic, isn't it?))

B-4 CASTING DEPARTMENT

Now In Production

"SHADOWS FROM THE FUTURE"

This issue of En Garde winds up four long years of regular publication. Considering the reputation the Editor acquired with the publication of NOVA, this is assuredly a startling and momentous fact. In fact, the Editor was so struck by this amazing event that it seemed fitting to celebrate in some fashion.

Wherefore, the next issue, the first issue of En Garde's FIFTH YEAR, will be a SUPER-DOOPER, ANNIVERSARY ISSUE!!! Don't miss it!

However, one embarks upon this project with some little trepidation. Not long ago, another member of FAPA announced for the next mailing, a "gala issue". It not only never appeared, but the member dropped out of FAPA---and Fandom too. Of course fans are too intelligent to be superstitious. Still as Campbell is so fond of saying, "But-------".

Anyway, it is to be hoped that the rest of the FAPA membership will cooperate by keeping their fingers crossed, and bolster our hope that En Garde doesn't meet with the same dire fate.

Watch for the Anniversary Issue of EN GARDE!!!

".....with jaundiced eye"

FANZINE YEARBOOK: The usual gratitude for such compilations, plus a little extra for the nice printed job you made of it.

AFTERTHOUGHT: I believe you have a point there, Doc, and it seems well-put.

LIGHT: An interesting batch of letters seems to make up a good share of this issue, and 'tis well. How much does Scripto base his analyses upon study of the person's handwriting, and how much on preconceived opinions of the person's character? Just curious.

BROWSING: Nice to see a bit of comment on the Mailing, and hope the department is enlarged in the future. Best Twelve Book Fantasies inspires the usual disagreement. One wonders whether and such list can ever have any real meaning aside from being an expression of one individual's opinion at the moment. Even polls on this subject must mean little. I doubt if you could find two fans who had read exactly the same group of fantasies. Then the order in which they were read, the age of the reader at the time of reading, etc., all unduly influence his choice. I'm afraid all the results of a poll indicate is that the average rating is thus and so, and said rating may have little connection with the actual quality of the volume. Britain Outside Fandom was much enjoyed.

A TALE OF THE 'EVANS: It is always interesting to read accounts of fen adventuring, but I believe I had much more fun making the trip in a car and being able to get right out any time I felt like it and rub Jack's nose in the scenery, than you could have viewing it from a train window. On the other hand, well, at least you didn't have tires to worry about and wrestle with repeatedly.

THE TIMEBINDER: No doubt the desire for personal security drives many to belief in a Supreme Being (or whatever one wishes to call it) on which one may lean. When one is unwilling, or unable due to lack of ability or lack of available data, to fathom some of the more perplexing mysteries of the universe, shrugging the whole thing off as the will of some Omnipotent Being, and no concern of our's, provides an easy way out. Inversely according to our feeling of competence and ability to cope with the universe is the need for security. When this need becomes sufficiently intense, one attempts to escape. In lower intensities, the desire may be mingled with other unsatisfied desires, and the escape be into fantasy (and other "escape literature"), alcohol, concentration on work or other occupation to the exclusion of all else, and such forms of periodic relief. When more intense, escape into religion and other forms of mysticism provides the necessary refuge. Finally, if the intensity becomes extreme, escape into some type of insanity is the general rule and provides an adequate haven.

So religion, and belief in a Supreme Being appears quite a natural thing. It's just the old matter of "supply and demand". The demand exists, due to quite explainable causes, whereupon either organized or individual religion is promptly supplied. The Atheist points with horror to the frightful things religion has done and the drag it has been on progress, but

overlooks the fact that religion is a symptom, not the cause. That is, the militant atheist so views it. The Agnostic, neither believing nor disbelieving, becomes little more than a fence-straddler. He can not be charged with lacking courage of his convictions because he avoids having any convictions, escaping instead into his in-between stand. Therefore it appears that the non-militant atheist takes the only valid stand. Realizing it is all a matter for individual decision, he attempts to consider it dispassionately.

(1) UNIVERSE ←—— NO CAUSE.
(effect)

(2) UNIVERSE ←—— CAUSE-1 ←—— CAUSE-2 ←—— CAUSE-3 ←—— ∞
(effect) (effect) (effect) (effect)

We are faced with a universe we are unable to explain more than partially, and that with no absolute certainty. To better examine our problem we generalize, and thus find two alternatives. In the first we have the Universe which is an effect. We postulate that there is no cause needed to explain it. Therefore, it follows that any concept of a Supreme Being that created it is superfluous. In the second we have the same Universe and offer Cause-1 to explain it. But Cause-1 is also an effect, so we bring in Cause-2, another effect, then Cause-3 and so on until we find ourselves with a regression to infinity. Feeling this unsatisfactory, suppose we try substituting Supreme Being for Cause-1. We can then either decide arbitrarily that Supreme Being explains everything and there is no need to think about it any farther, or we can be intellectually honest and admit that Supreme Being is an effect and still needs Cause-2 to explain it.....and so on. Why should a mind accept such abstract concepts as infinite distance, infinite time, or an infinite universe, yet balk at an infinite chain of cause and effect? Having seen so clearly that the interposition of Supreme Being really explains nothing, it obviously can be nothing but a symbol of a mind's refusal to reason beyond a certain point. Call it Supreme Being, Mind, Will, Power, or any other evasion one wishes. Embellish it with whatever other characteristics prove comforting. It still remains nothing but an escape from reality.

INSPIRATION: Glad to see Inspiration again, Lynn. On Time And Stuff was an interesting resume. Your wondering about the reactions of fans to atomic power may have to go unanswered for awhile as far as I am concerned. Like so many others, I'm still in the process of adapting my thinking to this new reality.

HORIZONS: Your suggestion for a really comprehensive fan index sounds decidedly interesting and worthwhile. But the necessary work involved seems utterly appalling. I shy away. As to number of pages published in fandom, I'm afraid counting up indicates only about 500 for me. Guess that doesn't even leave me in the running.

THE VOICE: In this matter of Canadian vs. U.S. money, I don't think suspiciousness has anything to do with it. It's just that the average American, business man or otherwise, doesn't like to be bothered with anything calling for special consideration. They <u>will</u> accept Canadian money at the current discount if it can't be avoided, but they'd much rather shove the responsibility for converting

it off onto the other guy. They're just lazy, not suspicious.

READER AND COLLECTOR: Glad to see this again---and such a nice large issue, too. Robert Butman's article was extremely interesting and I look forward to Chapter II. Most amusing thing in the whole mailing was your plaintive complaint about the "almost hiss". One can easily imaging your feelings.

FANTASY JACKPOT: Despite my dislike for most fan-fiction, I really enjoyed "The Stone". Also liked Mike Fern on the Hawaiian legends. Odd that he's never given us the benefit of his knowledge of such things before. The Outlaw Of Porn presented some interesting facts hitherto unknown, at least to me. Laney on The Histomap causes me to reassert my willingness to help on this project, although I'm hardly qualified to initiate it or take a leading part. Book reviews interesting, and welcome as usual. All in all, the mag is a nice job.

FANTASTS FOLLY: An account of minglings of British fen always makes good reading.

THE MAG WITHOUT A NAME: Lotsa pictures---always a good thing. Warner's article on Fandom provides another good summary, and again reveals how impossible it is to pin Fandom down to any accurate definition.

MILTY'S MAG (July): I love your TS publications, and they certainly give a swell running account of the Fan abroad. At least, from what you say, abroad is more accurate than at war.

PHANTAGRAPH (April): When Roger Bacon holds forth, he sounds for all the world like a thirteenth century Campbell Editorial.

PHANTAGRAPH (May): No comment.

FAN-DANGO: Curse Laney! By the time I recovered from the exhausting effort of beating him into putting out the issue of his FAPA mag, it was too late to get my own done. So I've been behind ever since-----and it's all his fault! Regarding the raising of the membership limitation, I'm in favor of it. Although it may be the stiffer activity requirments will make room for the present Waiting List fairly rapidly, we would soon reach a new equilibrium, and be faced with another long Waiting List. Applications have been coming in rather fast, too, and it may even be that the List never will decrease appreciably without raising the limit. Yeah, I favor uppin it a little.

WALT'S WRAMBLINGS: Bawdiness, books, and a new type binding for the replacement of staples.

ALLEGORY: This I enjoyed very much. It was well done, and provided a lot of fun picking out various fen from behind the disguise. Suggested much to consider.

SUSTAINING PROGRAM: In view of how you felt after completing this issue's section of reviews, let me help to justify your effort by

assuring you it was the most interesting and entertaining bunch of reviews I've read in a long time. And I like reviews. To me, one of the most intriguing features of FAPA is the opportunity to observe the reactions of the members to the ideas of others. Your comment on Evans' views regarding imagination reminds me that I've never set forth my side of that argument in print. What Jack, Ollie Saari and I maintained was that given a man with absolutely all knowledge, he would have nothing left to prevent boredom but to wallow in sensory debauch. EEE maintained that he would still have his imagination to fall back on. To make clear what we implied by all knowledge we went into more detail. The man knew every fact about the universe. But also, all knowledge must imply that he knew every possible, and impossible, inter-relation of two or more of these facts. In other words, he knew everything that had happened, or could happen, or couldn't happen. Still Everett maintained that through some mystic thing called imagination the guy could dream up new concepts to entertain himself. We held that imagination was only the building of new concepts from existing knowledge, and that our definition of all knowledge implied that the guy had already considered all possible combinations. Evans maintained that imagination could still dream up something new. In other words than his, he maintained that imagination was some undefinable ability to create a new concept from nothing. To sum it all up, Evans believes imagination is the power to create in the full, original meaning of the word---sort of an effect without cause other than the will to do so. The rest of us believe it consists only of dreaming up new combinations of known facts, or making inferences from them.

MILTY'S MAG (October): More of that fascinating running account. Hope you make the Pacificon, July 4, Milty. I want to be on hand to hear about item 7, and to see that Pompeian souvenir.

NONESUCH: Greetings, Ron. Glad to see your ( ) face amongst us. Nonesuch serves as a very fine introduction, and methinks you'll prove a very worthwhile member. Thanks too, for the book review.

PHANTASPHERE: The Sad Song Of The Spheres was excellent.

FANTASY AMATEUR: Yes, Juffus, I'm afraid you did overdo your endeavor to save money on this issue. Otherwise, very good.

IN MEMORIAM SARDONYX: We'll miss you, Russell. More, I doubt not, than many another who has dropped from fandom.

FAN-TODS: Chan Davis' Math Puzzle Dep't left me positively gasping, although I think I figured the answers to a couple of the new problems he offered. But I'm not going to be fool enough to present my answers for anybody's inspection. This preponderance of math in FAPA is doing one thing though---it's slowly bringing me to a point where I'll settle down to really studying math, and finally reaching a point where I can make all the FAPA math sharks squirm for a change............I keep telling myself. Your analysis of the fantasy content of the two mailings was swell. It seems to answer a lot of the question on that subject. And if, as I hope, you present the rest in the next F-T, it really should settle the matter pretty completely. How you can find time and energy to do it I'll never know, but I hope you do.

Fourth World Science-Fiction Convention Society

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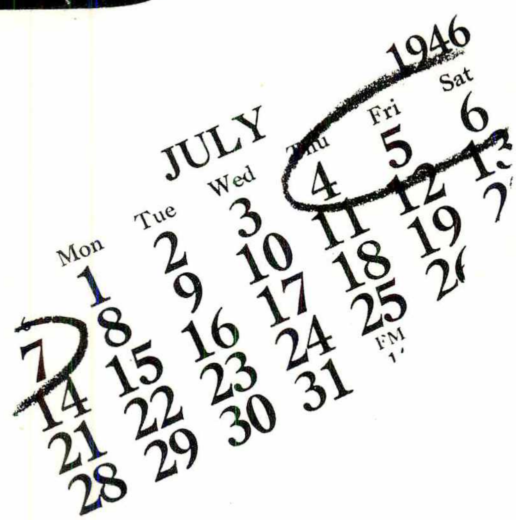

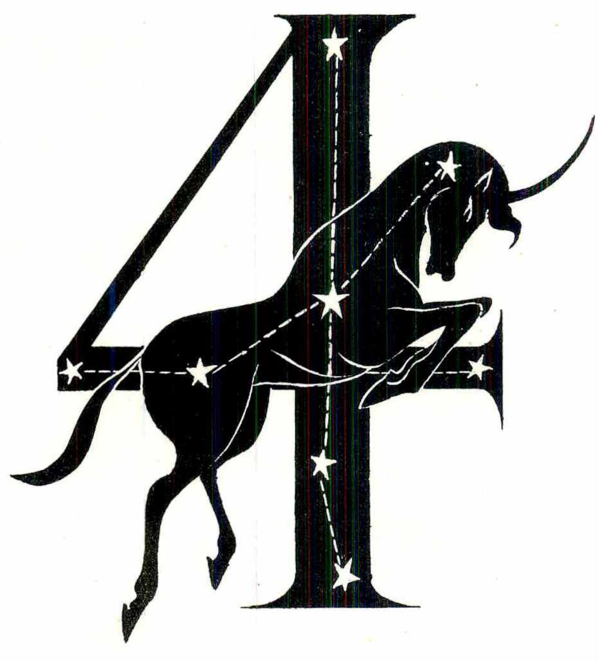

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PACIFICON

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HUBBA HUBBA

****

Or; Take Down That Service Flag, Mother, Your Son's In The ASTP.

By Milton Angstrom Rothman

The Galactic Fleet fired Joe Eager with ambition and thoughts of doing great things. His heart pounded when he took the letter from the pneumatic tube and read: "You will report ready for service in the Galactic Fleet 0800 12 Aug 2678."

He reported, mind filled with visions of heroism and a swift rise to the pinnacle of success and accomplishment.

The next day he was put in charge of the dishwashing machine at the mess hall.

Presently his training began. He went rapidly through his basic military training, and became a true Fleetman, straight, stern, physically fit, alert of mind, and brown of nose.

Suddenly, one day, the alarm rang through the quarters. The Galaxy had been invaded from the Outside!

Joe sat there, aghast. This meant war!

"This means war," he said.

"Yes," his buddies agreed. "This is it."

And they ran through the streets shouting: "Hubba Hubba!"

Joe had had two weeks instruction in the repair of gun sighting mechanisms (one week of which had consisted of the theory of optics), so he was considered competent to move to Santana, near Sirius, and become an instructor in the new training center which was to be opened there.

After two months of intensive preparation, which Joe occupied mostly with a strenuous program of sleeping, the new camp was ready for students. They came in droves, five and six at a time, and Joe taught them all that he could about fire control instruments, except for the week Classification sent him a half-dozen backward natives of Flitchikan who could neither read nor write. Joe found some difficulty in teaching them to calculate in four dimensions.

"But I'm afraid I'm getting into a rut," Joe said to his friends after a few months of this. "There must be something better than this."

So it was that when the opportunity came to take advanced training in sixth order forces at the University of Aldebaron, he leaped at the chance. Thoughts of learning great knowledge and becoming a high-ranking officer and doing great things swirled through his head.

Thus began a year of profound concentration and monastic application to studies. Tensor analysis, hypergeometry, fourth and fifth order mechanics. Outside in space, forces of unimaginable intensities slashed and burned as the battle continued to rage, while in the school, Joe Eager drooled deeply into the slavering piles of knowledge, keeping the vision of future magnificence firmly in mind.

Finally the course was completed, and he could relax once more. Now -- back to the Fleet and the hard active life. The war still raged. There was much to be done.

He was attached for a month to a casual outfit on Betelguese IV for reclassification and reassignment. He had a good time there amusing himself, reading books and shoveling gravel. His newly acquired knowledge was very useful in shoveling gravel.

His next stop was Communications Center on Procyon VI, where he dodged details for a month, took basic training again for three weeks, and then he was sent to Sirius XI for Communications training. There he learned practical work to supplement his theory.

Months sped swiftly by. The war continued to rage. Joe Eager learned more and more, and continued to think of the great work he was going to do when he had learned enough.

Finally the course was completed and he could relax once more. The camp on Procyon VI had closed in the meantime, so he took a spaship for Polaris III, stopping at home for a furlough on the way. There he had a wonderful time with his best friend's fiancee. His best friend was away on Antar IV at the time.

On Polaris III he had a stimulating period of sleeping, emptying garbage cans, sleeping, drilling, sleeping, seeing movies, sleeping, etc. The great visions still stirred in his mind, so he decided to apply for officer's training. He would become an officer and get out of all this.

So he filled out ten forms, and they bounced back because he had used the wrong abbreviation for Galactic Fleet Communications Service, Gamma Sector, Unit Command. So he filled out the ten forms again, and then discovered that his eyes had deteriorated so much because of his intense study at school that he could no longer pass the physical qualifications to become an officer.

Finally, joy, he was assigned to the crew of a battleship, and he entered a strenuous period of training and maneuvers. This was the life. Hard, real, the work brought the thoughts of great accomplishment into his mind once more.

Far out in space the war continued to rage. The invaders were

being driven out of their bases and back towards their own galaxy.

The time for Joe Eager and his ship to move on towards the battle zone rapidly approached. The crew worked feverishly to perfect its training. The final physical exams were given, and it was found that Joe Eager was allergic to the new synthetic foods that had just become standard for field use.

He was yanked out of the crew, and as he watched his old ship embark for battle, he boarded a liner for Betelgeuse IV. There he shoveled gravel for several more weeks, and was finally assigned to the Medical School at Alpha Centauri III. He studied to be a medical assistant for six months.

The day before he graduated the war ended and everybody ran around in the streets shouting: "Hubba Hubba!"

By that time Joe didn't give a damn what the hell happened.

SCIENCE IS COMMITTING ENORMITIES

By

Captain Donn Brazier

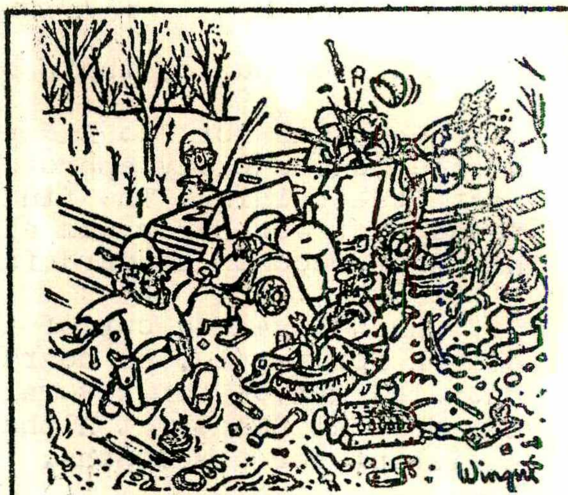
The following quotation from Aldous Huxley's "Time Must Have A Stop" could probably be utilized in an article on vivisection, but I'm too lazy to write it right now. I sort of wore out my enthusiasm for anti-vivisection when a sophomore in high school; built up a very impassioned and violent speech from material gathered from a semi-religious publication called THE GOLDEN AGE, and delivered the speech before the class.

" 'Cutting bits off frogs and mice, grafting cancer into rabbits, boiling things together in test tubes -- just to see what'll happen, just for the fun of the thing. Wantonly committing enormities -- that's all science is.' "

What do <u>you</u> say to that? One of Huxley's characters made the speech; he, himself, may or may not believe that.

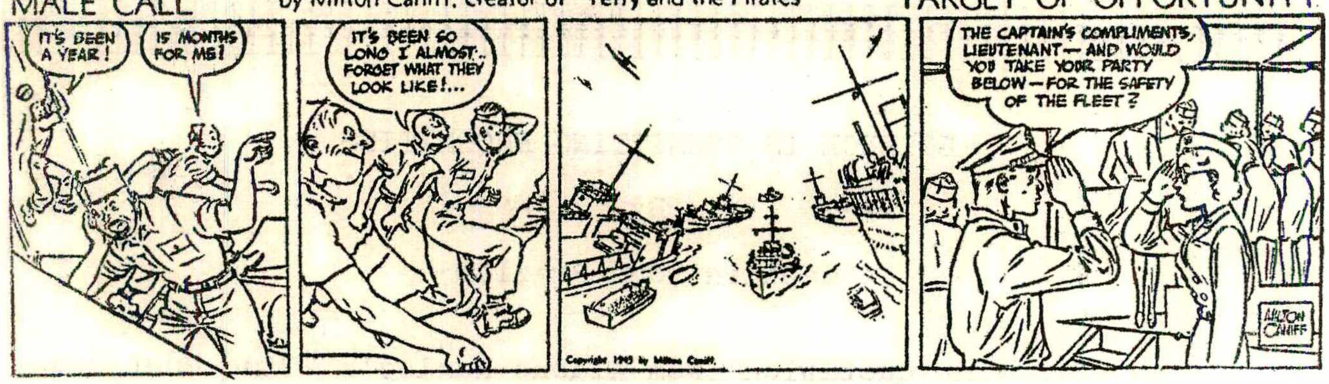
Is that all science is?

COMIC PAGE



"C'MAWN! We're movin' up!"

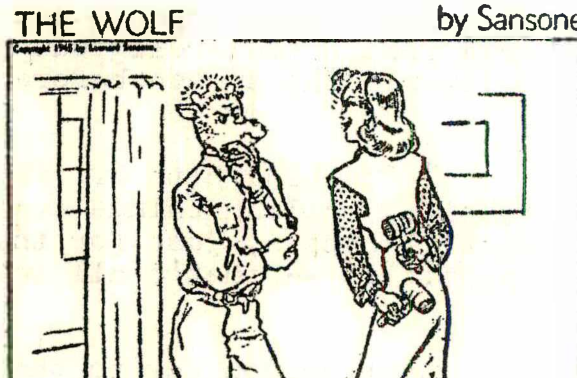
MALE CALL by Milton Caniff, creator of "Terry and the Pirates" TARGET OF OPPORTUNITY



THE WOLF by Sansone

"Oh dear! I should have warned you— Foo-Foo is terribly jealous!"

THE WOLF by Sansone



"Remember—guess the empty hand and you get a kiss!"

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SPACE AND HYPER-SPACE

By Chandler Davis

****

EDITOR'S FOREWORD:
This article is the first of a proposed series. It is our belief that the author has something to say, and is qualified to say it. The article deals with matters which should be of great interest to fans. However, as the reader gets into the article and begins to see the diagrams and bits of math, he will probably tend to shy away. The purpose of this foreword is to urge the reader to stick with it. The article contains nothing beyond the grasp of one willing to devote a little attention to what he is reading. With no more knowledge of math that a high-school course in algebra and geometry studied twenty years ago, and largely forgotten since, the editor was quite able to understand what follows without trouble. You can too! And your comments are anxiously solicited, and will be more than appreciated.

I

As much nonesense has been written in stefnal publications about this little matter of dimensions as about any phase of modern science. Pretty near the only correct statement that has seen print (if we except a few articles such as ASF's Unseen Tools) is that the world which our senses perceive contains three spatial dimensions. Most of the rest has been hogwash, or at best misleading half-truth. The situation arouses my crusading spirit to such an extent that I really wish I could sit down and go through vector space theory, Riemannian geometry, relativity, and some statistical mechanics right here. But I can't. I'll have to make the best of my limited time, patience, and knowledge, and simply give the subject a rough going over.

First I'll list the commonest boners. 1) Reference is frequently made to possible worlds existing "in the 4th dimension". This atrocity is committed less in bona fide s-f than in the semi-literate fields of fantasy and wierd stories (lay that pistol down, Searles!), but it sure is an atrocity. Anyone who has the least intuitive understanding of the concept of dimension can see that a BEM existing in the 4th dimension alone, that is, in only 1

dimension, would not be a BEM worthy of anybody's exorcising. In its own limited universe it might be quite fearsome, to be sure, since nothing in its path could possibly get out of its way, a 1-dimensional universe being the analogue of a single-lane highway; but I'm sure the invaders "from the 4th dimension" we read about are not envisioned by the authors as the super-thin earthworms they would actually have to be.

2) Any fan will tell you that Lovecraft was the very acme of erudition. Yet, in The Shadow Out Of Time (in a passage praised highly in a recent Fantasy Commentator) he makes the statement (I quote from memory) that "the recent researches of Professor Einstein indicate that time may be the fourth dimension". A mathematician hearing this feels his teeth set on edge, and may emit a strangled "Gaah!" (An incisive comment indicative of the high intelligence of mathematicians.) He will then try to clear things up by saying, "No, no, time is not the 4th dimension; but it is a 4th dimension"--thereby adding considerably to the unfortunate layman's confusion. Exactly what time is in relativity I'll try to explain later.

3) "What the geometry of hyper-space may be like, mathematicians of today cannot even guess." Statements like this appear occasionally in print, and people have spoken to me in conversation of "solving the problem of the 5th dimension" and other such crud. There is no problem of the 5th or any other dimension. Outside of not being able to visualize bodies in hyper-space so well, there is no essential difficulty in extra-dimensional geometry that is not met in the ordinary kind. Of course if you go into details you'll run into greater complexity (5 simultaneous quadratic equations in 5 unknowns present a rather knotty problem), but that's all. I may as well give here an illustration--one, by the way, which was sketched in the first part of And He Built A Crooked House. There is a certain class of regular bodies, which we may call in general "n-cubes", one body corresponding to each dimension. In 1-dimensional space, or, for short, 1-space, the body is simply a line segment; in 2-space, a square; in 3-space, a cube; or, generally, in n-space, an n-cube, where n represents any whole number. In 0-space, which is, of course, simply a point, since it cannot extend any distance at all in any direction or it would have dimension--the 0-cube is also a point. Now let's see what the relation is between these n-cubes, starting with the 0-cube. When the 0-cube is introduced into 1-space, it becomes free to move along the single straight line which constitutes this space. Let it do so, and let it somehow leave a trace of its passing in the space it traverses. After it has traveled a given distance, it will have described a line segment, or 1-cube. If this body is now introduced into 2-space and allowed to move perpendicular to its path through the same given distance, it traces out a square or 2-cube. Similarly, a square in 3-space moving perpendicular to its path (ie, to its plane), and oozing out ooze as it goes, eventually will have left behind it a solid cube of--er--ooze. Now you may not be able to conceive of the process's being carried one step further, by dropping the cube into 4-space and moving it perpendicular to all of its faces, it meanwhile oozing hyper-ooze; in fact I venture to say you will be entirely unable to

visualize it, unless you are Quintus Teal. But to show that we can tell a good deal about what would happen, let's consider another aspect of this cube-generation.

When the point moved, it generated, first, a line, and second, 2 points, the ends of the line, one representing its initial position and one representing its final position. Any point in any of the n-cubes, when the cube generates an (n+1)-cube, will similarly give rise to 1 line and 2 points. For example, the 4 points at the vertices of the square generate, in the cube, 8 vertices and 4 edges. (If you can't visualize this immediately, it'll help if you'll make the slight effort necessary to do so.) Similarly, when the line is moved out of its own path it produces 1 area, representing the space through which it traveled, and 2 lines, its initial and final positions; and the square's interior produces 1 volume (the interior of the cube) and 2 squares (2 of the cube's 6 faces). Extrapolating, quite legitimately, let's consider the hypothetical generation of the 4- by the 3-cube. Each of the cube's 8 vertices gives 2 vertices (making a total of 16) and 1 edge. Each of the cube's 12 edges gives 2 edges (making, with those obtained already, 32) and 1 area, or face. Each of the cube s 6 faces gives 2 faces (or an overall total of 24) and one <u>volume</u>.(The volumes are not the <u>interior</u> of the 4-cube, of course). The cube's interior volume gives 2 volumes (an overall total of 8) and 1 <u>4-volume</u>, the interior of the new figure. So in spite of not being able to visualize this figure, we have a pretty complete idea of its structure. Just to show the generality with which we can study n-cubes, I'll set down the complete box-score for all of them through the 5-cube,

	Vertices	Edges	Faces	Volumes	4-Volumes	5-Volumes
0-cube (point)	1	0	0	0	0	0
1-cube (line segment)	2	1	0	0	0	0
2-cube (square)	4	4	1	0	0	0
3-cube (cube)	8	12	6	1	0	0
4-cube (tesseract)	16	32	24	8	1	0
5-cube	32	80	80	40	10	1

We can further state with regard to all of these, and the higher n-cubes, that all their component edges, or faces, or volumes, etc, are congruent; that their component (n-1)-volumes (as the edges of a 2-cube, or the faces of a 3-cube, or the volumes of the 4-cube, etc.) occur in n parallel pairs, parallel having the same meaning in hyper-space as it does in 2- or 3-space; that the number of vertices of an n-cube is 2^n, and the number of n-volumes is 1; and many other facts.

All this relies considerable on your intuition, but should demonstrate that there is nothing mysterious about higher dimensions if we assume them (as you will grant we may reasonably assume in the absence of any other hypothesis) to be simply spatial dimensions behaving exactly like our own basic 3. Much more can be done, of course. I venture to say that the number of different regular polyhedra possible in 4-space could be determined--and proved-- by the same method used in 3-space. Whether this has ever been done or not I don't know, but if Stanley or somebody wants to try it it might prove interesting; though you might quite likely find that the only regular polyhedron is the tesseract.

II

Now for the general exposition of the subject. The natural place to start is with the concept of a vector space.

Those fans who were subjected at some time in their careers to college physics or analytic geometry will have some familiarity with the animal known as the vector. I hope they understood it at the time, but for those whose exposure was insufficient for the idea to take I'll try to explain it now. (The mathematically erudite should skip the next paragraph or two.)

My freshman physics instructor opened one of his first few lectures with the statement, "A vector is something which has magnitude and direction." The students remarked to each other behind the backs of their hands that the definition didn't mean a damn thing; the instructor gave them all a look of tacit agreement and went on. Actually, the definition does put it pretty well; but I think it's better to say a vector _is_ a magnitude and a direction. Visualize an arrow pointing in a given direction and with length equal to a given magnitude. Since position is not one of the things which determine a vector, the vector corresponding to this arrow is the same as that corresponding to a like arrow parallel to it at any point in space. It might represent, say, the velocity of a given particle at a given instant (its direction being that of the particle's motion, and its magnitude representing the particle's speed, in any appropriate units); and the vector's non-dependence on position is a reminder of the fact that 2 particles in widely separated positions may have the same velocity at the same instant. Or it might represent the "displacement" between 2 points, that is, the direction and distance of straight-line travel necessary to reach one from the other. Those two examples should be sufficient demonstration of the significance and importance of the concept of vector. Remember, you can move a vector parallel to itself all you want and it doesn't lose its identity; but if you change its direction or length it becomes a different vector altogether.

Now suppose we have a system of vectors (with any physical meaning whatever, it makes no difference) which is restricted to a plane. If we set up a system of rectangular coordinates in the plane and then move one of the vectors so that its base (the tail end of the arrow) is at the origin of the coordinates, the setup is like so (Fig. 1).

Fig. 1.

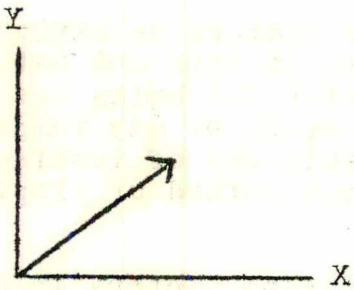
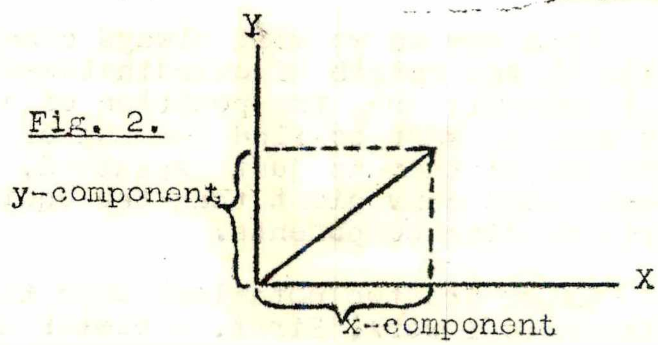


Fig. 2.



Finding the numerical values of the x- and y-coordinates of the head of the arrow, we have what are called the "components" of the vector (Fig. 2.) These components may be negative (Fig. 3, and Fig. 4.). Notice that, given the coordinate system, the components determine the vector without ambiguity. Also that if the length of a vector is multiplied by any number without its direction's being changed, the components are multiplied by the same number (Fig. 5.); and that if 2 vectors are "added" -- the tail of 1 placed at the head of the other as in Fig. 6, where the dotted vector is the sum of the two solid-line vectors--the respective components have only to be added in the ordinary way to give the components of the sum.

Fig. 3.

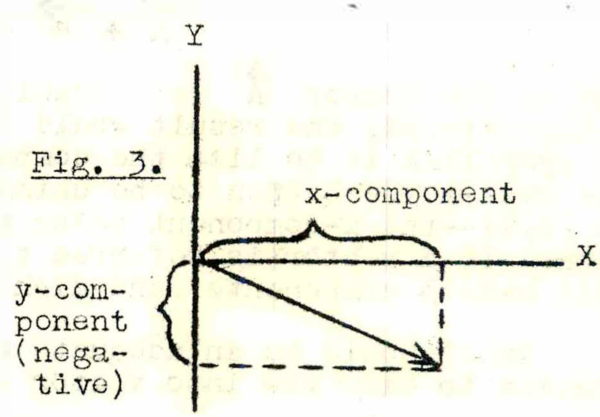
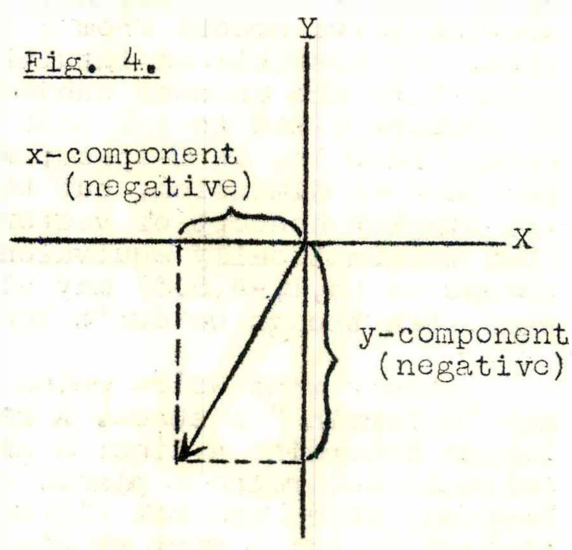


Fig. 4.



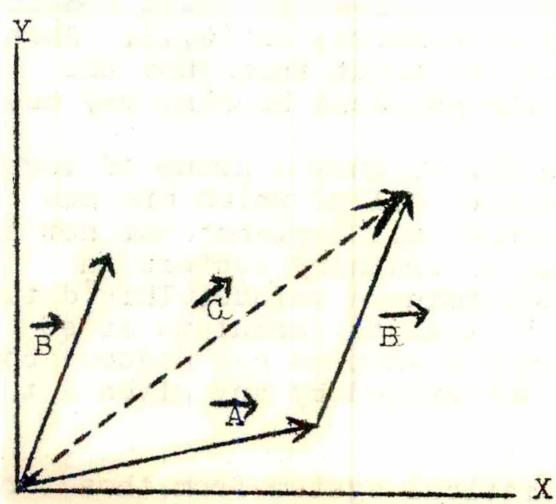
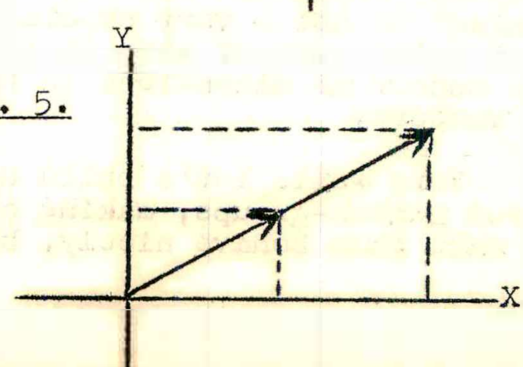


Fig. 6.

Fig. 5.



From now on we will always consider vectors as having their tails at the origin of coordinates--since it does not matter where they actually are, the position of a vector not being essential. But when we want to find the sum of 2 vectors we may temporarily move one of them as just explained, if this way of looking at it proves more convenient than the equivalent method of simply adding corresponding components.

There are two notations used to indicate vectors in ordinary mathematical work. First, a vector may be indicated by a capital letter with an arrow superscribed. Addition--the vector addition described above--is written as if it were ordinary addition. Thus the situation in Fig. 6, would be summed up

$$\vec{A} + \vec{B} = \vec{C}$$

And if the vector $\vec{A}$ were doubled in length without its direction's being altered, the result would be written $2\vec{A}$. The second method of symbolism is to list the components of the vector in the coordinate system you happen to be using. Thus the vector in Fig. 1, might be (2,1)--the x-component being written first as a matter of convention. If a vector is referred to 3 coordinate axes instead of 2, it will have 3 components, and might be written, for example, (3,1,2).

This should be an adequate sketch of the principles of vector algebra to ease you into vector spaces without too rude a shock.

Mathematicians don't feel satisfied with their understanding of a subject until they have developed it all by completely logical, non-intuitive proofs from a few simple assumptions depending as little as possible on intuition. It is necessary to do the same thing with the present subject. For in the exposition I have given of vectors I had to ask that you assume the vectors existed in the plane, that is, in 2-space; while we would like,not to start with the idea of dimension, but to get a clarification of the term from the simpler concept of vector, or better yet from the still simpler (but mathematically equivalent) groups of numbers, as (2,1). Such groups as (3,1,-4,6,0) may also be used; we treat them just the same even though we don't know what their physical meaning may be.

Here's what we're going to try to do. We know a group of vectors may "determine" a space. A group of vectors all of which are colinear determine a line; a group of vectors all coplanar but not all colinear determine a plane; and a group of ordinary vectors in 3-space, which are not all coplanar, determine a volume. This"determining" is not a very precise idea; we'd like to formulate it so that we're sure of what it is, and then see what we can deduce about the number of dimensions in the space determined by any given set of vectors.

Very well, let's build up a mathematical system from these ordered number-groups, making certain assumptions about them in order to make them behave nicely, but avoiding any preconception that they

do resemble sets of vector-components until we have shown that they do. We'll call 'em vectors, though, right from the start. All the vectors in any one system must be required to have the same number of numbers (if you see what I mean) in order that they may be made to behave like sets of components all related to the same system of coordinates. We'll phrase the definitions and so forth so that this number of coordinates may be anything we choose; but in giving examples we'll use 4, so that typical vectors would be $(1,0,0,0)$ and $(4,-3,1,\frac{1}{2})$.

We haven't yet defined addition of vectors, so we'll do it the obvious way. The sum of vectors $\vec{A}$ & $\vec{B}$ is defined as the vector obtained by adding corresponding components of A & B. Thus if $\vec{A}$ were $(0, 1, 5, 0)$ and $\vec{B}$ were $(\frac{1}{2}, -2, 0, 3)$, we would have

$$\vec{A} + \vec{B} = (0+\tfrac{1}{2},\ 1-2,\ 5+0,\ 0+3) = (\tfrac{1}{2},\ -1,\ 5,\ 3)$$

We will also define multiplication of a vector by a number in the way our previous discussion suggested. The product of vector $\vec{A}$ and number c is defined as the vector obtained by multiplying each of $\vec{A}$'s components by c. Thus if $\vec{A}$ is $(1, 0, -2, 3)$, then

$$2\vec{A} = (2, 0, -4, 6)$$
$$1\vec{A} = (1, 0, -2, 3) = \vec{A}$$
$$-2\vec{A} = (-2, 0, 4, -6)$$
$$0\vec{A} = (0, 0, 0, 0).$$

If you use these definitions on a vector with only 2 components, instead of 4, you find the results agree with those obtained from the ordinary definitions used for physical vectors above. Thus these number groups we've been calling vectors do, with the definitions we've made, act like vectors. Yet they're more general, for we need not limit the number of components to 3. We can, in fact, find what would happen if we did have 4 mutually perpendicular coordinate axes, by investigating our ordered number groups of 4 numbers.

III

The next thing on the program is the definition of "linear dependence", a concept whose importance will become apparent later. We say $\vec{A}$, $\vec{B}$, & $\vec{C}$ are linearly dependent if we can find numbers a, b, & c, none of which are zero, such that

$$a\vec{A} + b\vec{B} + c\vec{C} = (0, 0, 0, 0).$$

The same definition applies with obvious changes if you have a different number of vectors or a different number of components in each vector. Some examples are in order.

(1, 0, 2, 0), (-1, 3, -1, -2), & (-1, 6, 0, -4) are linearly dependent because

$$(1,0,2,0) + 2(-1,3,-1,-2) - (-1,6,0,-4) =$$

$$= (1,0,2,0) + (-2,6,-2,-4) + (1,-6,0,4) = (0,0,0,0)$$

Any vector is linearly dependent with any multiple of itself, for example

$$2(9,18,12,-3) + 3(-6,-12,-8,2) = (0,0,0,0)$$

A very simple example of vectors which are not linearly dependent is provided by (1,0,0), (0,1,0), & (0,0,1). Their mutual linear independence is obvious, but taken together with (1,1,1) or even (69,5,-732) they give a linearly dependent set. Any non-zero vector is linearly independent by itself, that is, you can't multiply it by a number different from zero and get a vector all of whose components are zero; and any zero vector (one all of whose components are zero) is linearly independent of any non-zero vector whatever.

Using the definition we have just set up, we can get down to the main point of just how a group of vectors may "determine" a space. Remember that so far this determining is something we understand only through geometry and intuition, so that we are unable to extend the concept to space of more than 3 dimensions. We met the same difficulty in our extending the idea of "vector" to extra dimensions, and we solved it there by setting up a completely abstract and general definition of vectors as ordered number-groups and then defining relations of addition and multiplication between them in such a way that they behaved, when they had 3 or fewer components, like ordinary vectors referred to coordinate systems having 3 or fewer perpendicular axes. Since it made no difference in our definitions if our vectors happened to have more than 3 components, we were confident that these definitions would lead to results in these difficult-to-imagine cases which would square with what we do know to be true in our universe.

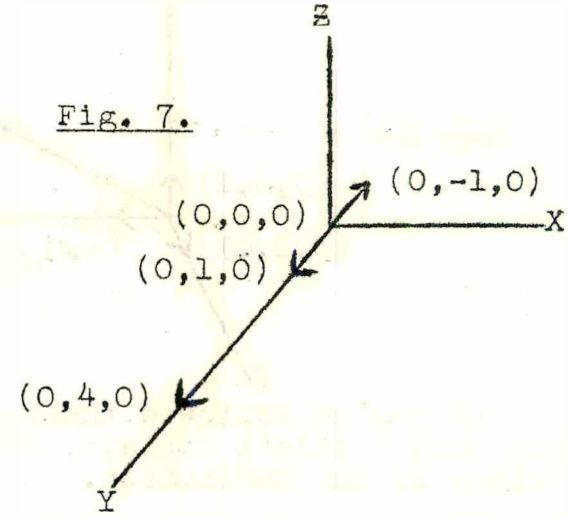
So we'll do the same thing here. Set up a definition, test it, and then, having tested it, apply it.

First of all, we will define a space as any class of vectors at all. If you will think it over you will see that this really is exactly what we meant by the term anyhow. (Pause while you think it over.) Then, on to the definition: The space determined by a set of vectors includes any vector which forms, with the given set or with any group of vectors from the given set, a linearly dependent group; it also includes (0,0,0) but does not include any further vectors. It doesn't matter whether the original set is linearly independent itself or not.

If I give some examples of this definition, and if you study them carefully, it'll make the testing of it that much easier. All the examples will use 3-vectors, so that you can visualize their significance intuitively by thinking of a coordinate system with 3 axes.

The vector (0,1,0) determines a space which includes any 3-vector whose first and last components are zero, but does not include any other vector. Exactly the same space is determined by the two 3-vectors (0,1,0) & (0,58,0). (0,1,0) & (0,0,0) also determine this space. See Fig 7; this figure, and those which follow it, are really views of 3-dimensional figures.

Fig. 7.



(1,0,0) & (0,1,0) determine a space which includes any vector whose last component is zero. For example, (5,-4,0) is included, since

$5(1,0,0)-4(0,1,0)-(5,-4,0) =$

$= (0,0,0)$

and (6,0,0) also, since

$6(1,0,0) - (6,0,0) = (0,0,0)$

But no vector with last component different from zero is included; because no sum of multiples of the determining vectors will have its last component different from zero, and therefore any such sum added to any multiple of the vector in question will give a sum with a non-zero final component, hence cannot give (0,0,0) as required. See Fig. 8.

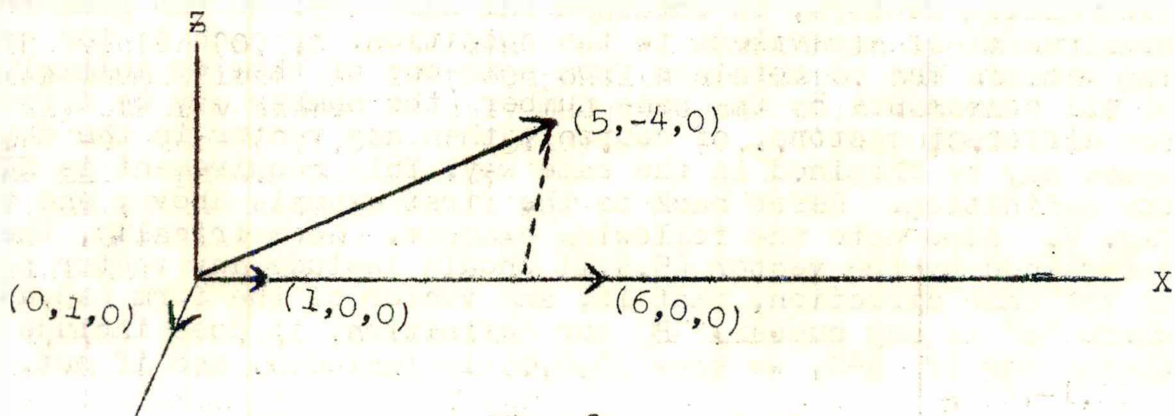


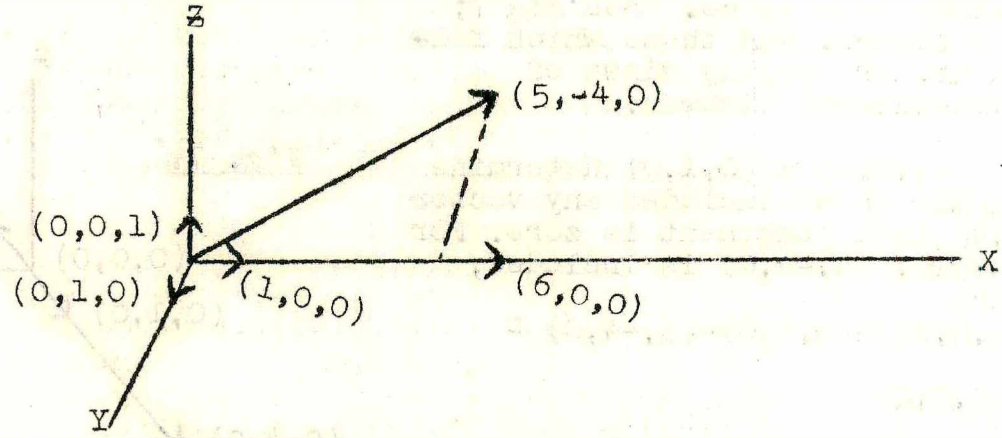
Fig. 8.

Now observe that all four of (1,0,0), (0,1,0), (5,-4,0), and (6,0,0), taken together, determine the same space as do the first two taken by themselves. The same space is determined by any three of them; and by any two of them, except (1,0,0) & (6,0,0). The space determined by this pair would not, for example, include (0,1,0).

The space determined by (1,0,0), (0,1,0), & (0,0,1) includes all 3-vectors. (1,3,0), (2,-1,0), & (1,3,4) will serve just as well. So only three vectors are necessary to determine this space. Yet we can easily find sets of more than three vectors which do not

determine all of it. In fact we have already done so(see the last paragraph); (0,0,1) is not included in the space determined by (1,0,0), (0,1,0), (5,-4,0), (6,0,0). See Fig. 9.

Fig. 9.



If you've followed these examples and figured out those of them that I didn't explain, you won't have any trouble with the testing of the definition.

First, if our definition agrees with geometry, any number of colinear vectors should determine the line along which they lie. But, since colinear vectors are simply vectors having the same direction (or exactly opposite directions) you can obtain one from another by multiplying all its components by the same number. Any vector in the determined line, being by geometry colinear with the determining vectors, is obtained the same way. So the geometrical requirement is equivalent to the condition: If each of the determining vectors can be obtained from some one of them by multiplication of all components by the same number (the number may be different for different vectors, of course), then any vector in the determined space may be obtained in the same way. This requirement <u>is</u> met by the definition. Refer back to the first example above, and to Fig. 7. Also note the following example. Geometrically, the space determined by the vector (5,4,1) should include any vector pointing in the same direction, that is, any vector of the form (a5,a4,a), where "a" is any number. By our definition, it does include all these; for if a=0, we know (0,0,0) is included, and if not, then

$$a(5,4,1) - (a5,a4,a) = (0,0,0)$$

and (a5,a4,a) is linearly dependent with the determining vector.

The second geometrical requirement is that any number of coplanar, but not colinear, vectors must determine the plane in which they lie. This requirement, too, we'll have to investigate geometrically to see just what it means before we can apply it. Suppose we have any two (or more) coplanar vectors. Then let's find out first whether the sum of any pair lies in the same plane. You will remember that to find the sum we may move one of the two so that its tail coincides with the head of the other. Their sum is then the vector pointing from the origin to the new position of the

moved head. But in moving the vector we kept it parallel to its original position (otherwise it would have become a different vector), and in its new position it passes through a point--the other vector's head--which is in the plane of the two original vectors. Therefore it is still in this plane, its head is still in the plane, and the sum vector, having both tail and head in the plane, is also in it, and our assumption is proved correct. Now in addition it is self-evident that any multiple of any of the group of coplanar vectors with which we started is in their plane. From these two facts it follows that any <u>sum of multiples</u> of the coplanar vectors is in the plane also! Knowing this makes our job very easy. For it means that any vector in the plane of our determining set is expressible as a sum of multiples of the set. So now to test our definition we have only to show that it includes in the determined space all sums of multiples of the set and excludes all other vectors.

Suppose the set is $\vec{A}$, $\vec{B}$, and $\vec{C}$, and let $\vec{D}$ be any vector in the plane, so that some numbers a, b, & c can be found so that

$$\vec{D} = a\vec{A} + b\vec{B} + c\vec{C}$$

Then it's obvious that

$$a\vec{A} + b\vec{B} + c\vec{C} - \vec{D} = (0,0,0)$$

so that the definition checks here too. And it's equally obvious that if $\vec{D}$ were not expressible as in the first of these equations then no equation such as the second would be found to hold. So we are all set.

The next part of our test I'll let you work out for yourself. If $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ are 3-vectors which are not coplanar, they should determine the space of all 3-vectors. If you can prove that any 3-vector can be expressed as a sum of multiples of these arbitrary non-coplanar vectors, then you can complete the testing of the definition much as I have just done for the case where the set is coplanar. If you're good at algebra the proof is a snap; if you are not, content yourself with verifying it intuitively.

So now the definition has been weighed in the balance and found not wanting. By now you should have an inkling of how we're going to go about obtaining our main result. We were trying, you recall, to define the "dimension" of an arbitrary space in a way which would depend only on the components of the determining vectors.

We have already seen that many different sets may determine the same space. It should be plausible to you that in order to define the dimension of a space we would like to find a determining set for it which is as simple as possible. Specifically, we would like to get as <u>few</u> vectors in the set as possible. OK, let's do.

It is a fact that if any group of vectors from the set is linearly sependent, then one of this group may be eliminated from the set and the same space will still be determined. If one of the

group is a zero vector; you can see from the definitions I have given that it can be eliminated right away; so we'll just take the case where they're all non-zero. Suppose

$$\vec{A},\ \vec{B},\ \vec{C},\ \vec{D}, \text{ and } \vec{E}$$

determine a space and also

$$a\vec{A} + b\vec{B} + c\vec{C} + d\vec{D} = \vec{O}$$

(where $\vec{O}$ represents the zero vector), or

$$\vec{D} = -\frac{a}{d}\vec{A} - \frac{b}{d}\vec{B} - \frac{c}{d}\vec{C}$$

Then if any vector $\vec{V}$ is included in the space by virtue of a relation

$$a'\vec{A} + b'\vec{B} + c'\vec{C} + d'\vec{D} + e'\vec{E} + v'\vec{V} = \vec{O}$$

it will also be true that

$$\vec{O} = a'\vec{A} + b'\vec{B} + c'\vec{C} - d'\left(\frac{a}{d}\vec{A} + \frac{b}{d}\vec{B} + \frac{c}{d}\vec{C}\right) + e'\vec{E} + v'\vec{V}$$

$$= (a' - d'\frac{a}{d})\vec{A} + (b' - d'\frac{b}{d})\vec{B} + (c' - d'\frac{c}{d})\vec{C} + e'\vec{E} + v'\vec{V}$$

and $\vec{V}$ will still be a member of the space after $\vec{D}$ is eliminated from the determining set. This proof can be worked backward to show that no vector which was in the space when $\vec{D}$ was in the set will be in it when $\vec{D}$ is eliminated, but I won't bother going through that here. It's simple enough anyway. Furthermore, you can see that the same proof would hold if we had more (or fewer) than five members in the determining set, and if we had more (or fewer) than four members thereof linearly dependent.

So if we were studying, for example, the space determined by (0,1,4,0), (0,0,0,0), (-1,2,-2,1), & (-2,5,0,2), we'd do well to drop $\vec{O}$ from the determining set, and also (-1,2,-2,1), since

$$(0,1,4,0) - (-2,5,0,2) - 2(-1,2,-2,1) = (0,0,0,0)$$

As to the space determined by (0,0,0,0) & (0,0,0,0), we just follow our rule dropping zero vectors, and, with a twist of the wrist, eliminate everything! This seems a little arbitrary, no doubt, but we have to have some kind of rule to cover this special case, so we simply say that the space determined by no vectors at all shall contain $\vec{O}$ and nothing else; and our blood-purge of the determining set is justified.

In the example which accompanied Fig. 8, we observed that the

space given by (1,0,0), (0,1,0), (5,-4,0), & (6,0,0) was the same as that given by (1,0,0) & (0,1,0). This fact can be obtained from the rule we have now developed. First we eliminate (6,0,0) from the set on the basis that

$$6(1,0,0) - (6,0,0) = (0,0,0)$$

From the resulting set of three vectors we drop (5,-4,0) because

$$-5(1,0,0) + 4(0,1,0) + (5,-4,0) = (0,0,0)$$

and we are left with (1,0,0) & (0,1,0), as observed before.

Now at last we are ready to define dimension. It's a beautifully simple definition, checks easily with our intuitive ideas, and is useful in situations where the latter would be entirely useless. Definition: the dimension of a space is the smallest number of vectors which can determine the space.

We compare this with our intuitive notions by considering 3-vectors, just as we have done with other definitions. This time, however, it's no trouble at all. First, is a 0-space determined by a set having no vectors at all? Well, the only 0-space or point we can have in vector space is the vector that doesn't go anywhere, $\vec{0}$; and we have seen that the space containing only $\vec{0}$ is determined by no vectors. Second, is a 1-space determined by a set having one vector? Sure: a set of collinear vectors determine a line, as we saw while testing the definition of "determine"; and it's obvious that all but one of a set of colinear vectors can be dropped, by linear dependence. Thirs, is a 2-space determined by a set having two vectors and no fewer? We know that two is the smallest number of vectors that can be coplanar without being colinear; so score another for our side. Last, is a 3-space determined by a set having three vectors and no fewer? Once again, three is the smallest number of vectors you can have that won't necessarily be coplanar. And there we are. The geometrical nature of the checks we have just made is nothing against them, since we have already checked our definition of "determine" against geometry.

IV

This section will be a summary of what we've achieved, and an abstract of what is yet to come.

We've defined dimension. That's the main thing. Assuming nothing about 4-space except that it behaves just like our 3-space, and that no particular direction in it is singled out by any physical peculiarity as <u>the</u> 4th dimension, we can tell a hell of a lot about it. Imagine yourself a 4-man (I do not mean a supervisor in a mill). Imagine yourself, further, a 4-astronomer. Now since there are four dimensions in your space, the coordinate system you will use to describe the positions and velocities of your heavenly 4-bodies will have four mutually perpendicular axes. (This seems obvious, but if you're a fiend for mathematical precision you may want to check it by the definition of "dimension" and the other results of sections II & III.) Suppose that one day, peering through

your 4-telescope, you observe some mysterious meteorites appearing out of noplace, and suspect, being a 4-fan, that they have come---out of the dimensions! And are not 4-meteorites at all but 3- or even 2-meteorites. What do you do? You measure their positions and velocities instantaneously (your science being far more advanced than ours), before they've had time to be pulled out of their paths by your 4-sun's attraction. Then you lift your 4-hand and say, "Let there be vectors!" and there are vectors. You take the position of one of the meteorites as your origin of coordinates; put your perpendicular axes through it; and, using this system of coordinates, find the components of, first, the vectors representing the distance and direction from this meteorite to each of the others, and second, those representing the velocities of all the meteorites. The space determined by these vectors must have the same number of dimensions as the space from which the meteorites came. Far into the night your 4-pencil races feverishly over your 3-paper. (Yes, I said 3-paper---we use 2-paper, don't we?) You are finding out how many of the determining set you started with can be eliminated. If you end up with four left in the set, you'll know you were wrong and that your 4-Astoundings have been too much on your mind. If you end up with fewer than four in the set, you still won't be able to convince the Royal 4-Academy that you're anything but a mental case, but that's beside the point. The point is that I know exactly what mathematical steps you would have to take to verify your wacky supposition, even though I've never been in 4-space myself.

We've achieved more than this. We can not only make statements about 4-space, we can make statements about space of an arbitrary number of dimensions. The best example is the general case of the fact I suggested you might prove a paragraph or so ago. The general theorem is this: the space of all vectors which can be described by a given coordinate system has as many dimensions as there are axes in the system. The proof is simple. We include in our determining set one vector along each of the axes; these vectors point along the positive directions of the axes, and are one unit long. Thus if we are using 5-vectors, our determining set would be

(1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), & (0,0,0,0,1).

These do actually constitute a determining set for the whole space; for to express a vector $\vec{A}$ as a sum of multiples of them, we simply multiply the first by $\vec{A}$'s first coordinate, the second by $\vec{A}$'s second coordinate, etc, and add the results. And we can't leave out one of this set, for then we couldn't take care of vectors having a non-zero component along the axis whose representative we had eliminated. Then since the number in our set is equal to the number of axes, the theorem is proved.

A little caution to the mathematical neophytes, and an apology to the sophisticates. In sections II & III, I did not use completely kosher terminology, and some of my definitions and proofs would have given at least one Harvard prof the screaming meemies. (I'm talking about D V Widder, in case any of you know him.) No, I was far from rigorous, and very far from conventional in my presentation. But I think you will find no mistakes, unless Ashley slips

some in in the mimeography, and I also think I got the idea across. ((If he only knew! --AA))

In the second installment of this article I'm going to discuss the following: The use of extra-dimensional space in mechanics. Ditto in statistical mechanics. Time as a 4th dimension. Infinite-dimensional space (You heard me), and maybe some more if I have time. This will appear in a later En Garde, I hope the next. The material will not depend directly on the details of vector space theory, but you should keep in mind the general ideas, especially (1) the equivalence of an ordered number-group with a vector as far as mathematics is concerned, (2) the close similarity in mathematical treatment of higher dimensions to the treatment of three or fewer dimensions, and (3) the fact that dimension can be rigorously defined. This should not be too much for you to carry over three months.

Ah-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-h-

QUOTE DEPARTMENT

By Donn Brazier

Much has been written in our literature of time; therefore, the following quotation from Gustaf Stromberg's "The Soul Of The Universe" may be old stuff to you. Perhaps, though, you haven't been introduced to the idea in quite those words.

"If a man had no memory, he would probably never arrive at a conception of time. For him there would apparently be only one moment, 'now'. Further, if the world around us were not changing, if man saw the same picture, heard the same sound and had exactly the same sensation and the same emotion all his life, he would have great difficulty in conceiving any sequence of time, since one part of the 'arrow of time' could not be distinguished from another part."

"Every notable advance in technique or organization has to be paid for, and in most cases the debit is more or less equivalent to the credit." Some character in Huxley's Time Must Have A Stop says so, and keeps repeating as if in refrain: "Backwards and downwards." An example cited to illustrate the point: universal education is teaching everybody to read the billboards!

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The End

of four long years of hard work, and......the beginning of another!